# INTERACTION OF A JET WITH A PLANE BOUNDARY LOCATED NORMAL TO THE JET AXIS

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The interaction of plane and axiosymmetric subsonic jets with a plane boundary located normal to the jet axis is studied. A numerical solution for the equations of motion and energy is obtained, assuming stationary flow and constancy of liquid properties.

The interaction of a jet with a plane boundary located normal to the jet axis is of undoubted interest, because of the large number of practical applications of such a situation. The region of interaction with the boundary may be divided into three sections:

1) the free jet region, according to experimental data, corresponds to distances above the boundary of the order of the width (diameter) of the nozzle output section;

2) the direct interaction area, characterized by significant pressure gradients both along and normal to the boundary;

3) the area of flow along the wall, with practically no pressure gradient.

The free jet region and region of flow along the wall have been studied sufficiently in [1, 2]; the flow of the jet in the neighborhood of the braking point has not been examined deeply at present. Empirical relationships for friction and thermal flux in this region have been obtained [3-9]. A wide divergence between the data of different authors, as well as large disagreements between experimental data and calculated results are based on the boundary layer equations characteristic of these relationships. In [10, 11] these deviations were explained as the influence of turbulence peculiar to the free stream upon friction and heat transfer between jet and boundary. According to our data there is not, at present, a sufficiently complete theoretical analysis that would permit evaluation of the effect of jet turbulence on friction and heat transfer. As a rule, either the flow of a nonviscous jet in the region of spreading over the boundary, or the flow of a viscous jet in the vicinity of the braking point is analyzed, the latter based on the solution of approximate boundary layer equations with boundary conditions obtained by experiment [12-14]. In the present study we have attempted to examine the flow in the region of interaction of a subsonic jet with a boundary on the basis of solution of exact motion and energy equations with subsequent evaluation of the effect of jet turbulence on friction and heat transfer with comparison of the results obtained with experimental data.

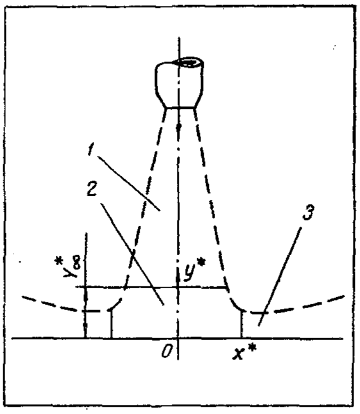


Fig. 1. Interaction of jet with boundary wall in regions of: 1) free jet; 2) interaction; 3) flow along wall.

Flow and Heat Transfer in the Vicinity of the Braking Point of a Subsonic Jet. We will examine the stationary interaction of a plane or axisymmetric jet with an infinitely large boundary wall located normal to the jet axis, $y^* = 0$, with a braking point $x^* = y^* = 0$ (Fig. 1). We assume that the influence of the wall on the flow is negligibly small at distances $y^* \geq Y^*_\infty$. From experimental data on a plane jet in the vicinity of the braking point [7] and an axisymmetric stream [12] it it known that the change in the velocity component normal to the

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boundary along the axis of symmetry of the jet ($x^* = 0$) may be represented approximately in the same manner as for a uniform potential flow, flowing onto a plane surface: $v^* = -\beta y^*$ (plane flow in vicinity of braking point); $v^* = -2\beta y^*$ (axisymmetric flow in vicinity of braking point).

The linear rule for change in the normal velocity component over the limits of the viscous mixing layer of the wall excludes a smooth juncture of the solutions for the free jet region and the region of interaction. Therefore, this analysis is valid only in the direct vicinity of the wall. We assume that the velocity and temperature profiles in the boundary layer of the free jet ($y^* \geq Y^*_\infty$) are described by the Schlichting and Taylor profiles respectively:

$$\frac{u^*}{u^*_m} = \left[1 - \left(\frac{x^* - x^*_0}{\delta^*}\right)^{\frac{3}{2}}\right]^2, \quad x^*_0 \leqslant x^* \leqslant R^*_b, \tag{1}$$

$$\frac{\Delta T^*}{\Delta T^*_m} = \sqrt{\frac{u^*}{u^*_m}}, \quad x^*_0 \leqslant x^* \leqslant R^*_b, \tag{2}$$

where $x^*_0 = 0$ for the completely developed jet; $\delta^*$ is the thickness of the boundary layer of the free jet; $u^*_m = U^*_e$, $T^*_m = T^*_0$ for the jet interacting with the boundary at the limits of the original portion of the jet.

Basic Equations. We introduce the following dimensionless variables:

$$(u, v) = \frac{(u^*, v^*)}{\sqrt{\beta\nu}}; \quad (x, y) = (x^*, y^*)\sqrt{\frac{\beta}{\nu}};$$

$$\Omega = \frac{\Omega^*}{\beta}; \quad T = \frac{T^*(x^*, y^*) - T^*_w}{T^*_m - T^*_w}.$$

Writing the equations of motion and energy in these new dimensionless variables, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \xi\frac{u}{x} = 0,$$

$$u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} = \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} + \xi\left(\frac{u}{x}\Omega - \frac{\Omega}{x^2} + \frac{1}{x}\frac{\partial \Omega}{\partial x}\right),$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\xi}{x}\cdot\frac{\partial T}{\partial x}\right], \tag{3}$$

where $\xi = 0$ for a plane jet; $\xi = 1$ for an axisymmetric jet.

The Plane Jet. Since the profile of the normal velocity component in the interaction region must, in view of our assumptions, tend to the velocity profile of the free jet at the border of the interaction region $y^* = Y^*_\infty$, we can write:

$$v^* = -\frac{U^*_e}{Y^*_\infty} y^*, \quad 0 \leqslant x^* \leqslant x^*_0;$$

$$v^* = -\frac{U^*_e}{Y^*_\infty}\left[1 - \left(\frac{x^* - x^*_0}{\delta^*}\right)^{\frac{3}{2}}\right]^2, \quad x^*_0 \leqslant x^* \leqslant R^*_b$$

or

$$v^* = -\beta F'(x^*) y^*, \tag{4}$$

where $\beta = U^*_e / Y^*_\infty$; $F'(x^*) = 1$, $0 \leq x^* \leq x^*_0$, $F'(x^*) = [1 - \{(x^* - x^*_0)/\delta^*\}^{3/2}]^2$, $x^*_0 \leq x^* \leq R^*_b$.

From the continuity equation it follows that

$$u^* = \beta F(x^*), \tag{5}$$

where $F(x^*) = x^*$, $0 \leq x^* \leq x^*_0$,

$$F(x^*) = x^*\left[1 - \frac{4}{5}\left(\frac{x^* - x^*_0}{\delta^*}\right)^{\frac{3}{2}} + \frac{1}{4}\left(\frac{x^* - x^*_0}{\delta^*}\right)^3\right], \quad x^*_0 \leqslant x^* \leqslant R^*_b.$$

In dimensionless form, Eqs. (4), (5) are written as

$$u = F(x); \quad v = -F'(x)\, y. \tag{6}$$

These equations describe the velocity distribution in the region of interaction of the jet and wall outside the zone of viscous mixing. For the flow in the viscous mixing zone we have

$$u = F(x) f'(y); \quad v = -F'(x) f(y). \tag{7}$$

The dimensionless temperature distribution in the free jet is described by

$$T = 1, \quad 0 \leqslant x \leqslant x_0,$$
$$T = \left[\sqrt{F'(x)} + \alpha\left(\sqrt{F'(x)} - 1\right)\right], \quad x_0 \leqslant x \leqslant R_b.$$

In the viscous mixing zone the temperature profile is

$$T = G(x)\,\theta(y), \tag{8}$$

where $G(x) = 1,\ 0 \le x \le x_0,\ G(x) = \sqrt{F'(x)} + \alpha(\sqrt{F'(x)} - 1),\ x_0 \le x \le R_b$.

Substituting the expressions derived for velocity and temperature in the viscous mixing zone in Eq. (3), we obtain the turbulence transfer equation

$$f^{IV} + B_1(ff''' - f'f'') + B_2 f'' - B_3 ff' + B_4 f = 0, \tag{9}$$

where $B_1 = F'(x)$; $B_2 = 2F''(x)/F(x)$; $B_3 = [F'''(x) - F'(x)F''(x)]/F(x)$; $B_4 = F^{IV}(x)/F(x)$, and the energy equation,

$$\theta'' + \Pr(A_1 f\theta' + A_2 f'\theta) + A_3\theta = 0, \tag{10}$$

where $A_1 = F'(x)$; $A_2 = -F(x)G'(x)/G(x)$; $A_3 = G''(x)/G(x)$.

Axisymmetric Jet. In analogy with the plane jet, we can write

$$v^* = -2\beta F''(x^*)\,y^*,$$

where $\beta = U^*_e/2Y^*_\infty$; $F''(x^*) = 1,\ 0 \le x^* \le x^*_0,\ F''(x^*) = [1 - \{(x^* - x^*_0)/\delta^*\}^{3/2}]^2,\ x_0^* \le x^* \le R^*_b$. Then

$$u^* = 2\beta\left[F'(x^*) - \frac{F(x^*)}{x^*}\right].$$

In dimensionless form

$$v = -2F''(x)\,y; \quad u = 2\left[F'(x) - \frac{F(x)}{x}\right]. \tag{11}$$

For the flow in the viscous mixing zone we have

$$v = -2F''(x)\,f(y); \quad u = 2\left[F'(x) - \frac{F(x)}{x}\right]f'(y). \tag{12}$$

The temperature distribution in the free jet has the form

$$T = 1, \quad 0 \leqslant x \leqslant x_0,$$
$$T = \sqrt{F''(x)} + \alpha\left(\sqrt{F''(x)} - 1\right), \quad x_0 \leqslant x \leqslant R_b. \tag{13}$$

For the viscous mixing zone

$$T = G(x)\,\theta(y), \tag{14}$$

where $G(x) = 1,\ 0 \le x \le x_0,\ G(x) = \sqrt{F''(x)} + \alpha(\sqrt{F''(x)} - 1),\ x_0 \le x \le R_b$. Substituting Eqs. (12), (14), in Eq. (3) we obtain the turbulence transfer equation

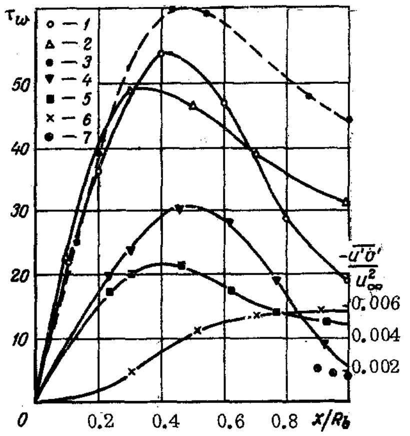


Fig. 2. Distribution of friction along wall in interaction region. $X^*/d^*_e = 8.0$ 1) calculation (plane jet); 2) calculation (axisymmetric jet); 3) experiment [3] (axisymmetric jet, $X^*/d^*_e = 18$); $X^*/d^*_e = 2.0$: 4) calculation (plane jet); 5) calculation (axisymmetric jet); 6) apparent turbulent flow tension; 7) experiment (jet along a wall [15]).

$$f^{IV} + B_1 ff''' + f''(B_2 f' + B_3) + B_4 ff' + B_5 f = 0, \tag{15}$$

where $B_1 = 2F''$; $B_2 = 2[(2/x)(F' - F/x) - F'']$; $B_3 = 2F'''/(F' - F/x)$; $B_4 = 2/(F' - F/x)[f'''(F'' + F'/x + F/x^2) - F^{IV}(F' - F/x)]$; $B_5 = [F^V + F^{IV}/x - F'''/x^2]/(F' - F/x)$; and the energy equation

$$\theta'' + 2\Pr A_1 f\theta' + (A_2 \Pr f' - A_3)\theta = 0, \tag{16}$$

where $A_1 = F''$; $A_2 = -2(F' - F/x)G'/G$; $A_3 = (F' - F/x)G''/G + (1/x)G'/G$.

The boundary conditions for both plane and axisymmetric jet are written as

$$f(0) = f'(0) = \theta(0) = 0; \quad f'(\infty) = \theta(\infty) = 1; \quad f''(\infty) = 0.$$

It should be noted that for $0 \le x \le x_0$, the turbulence transfer and energy equations have a form analogous to that for a uniform flow flowing onto a plane boundary [16].

Numerical Solution. A self-similar solution of the turbulence transfer and energy equations in the region $x_0 \le x \le R_b$ is impossible due to the dependence of the flow parameters on the x-coordinate. Therefore, solutions of Eqs. (9), (10), (15), (16), were obtained by the method of local similarity. For $0 \le x \le x_0$ a solution is known and has been tabulated [16]. The equations were solved by the Runge–Kutta method on a Minsk-2 digital computer with an accuracy to $10^{-3}$. Calculations were performed for the following initial data: $U_e^* = 20$ m/sec (plane stream); $U_e^* = 40$ m/sec (axisymmetric stream), $d_e^* = 10$ mm; $\nu = 20 \cdot 10^{-6}$ m²/sec; Pr = 1.0.

To calculate the effect of the relationship between temperature of the jet and of the boundary, two cases were examined: 1) hot jet, cold boundary ($\alpha = 0$); 2) cold jet, heated boundary ($\alpha = -1.0$).

The constant $\beta$ of the flow velocity in the vicinity of the braking point is determined by the following method.

1. Plane jet. According to the data of [7], $\beta$ is approximately equal to the ratio $U_e^*/d_e^*$. Then, in accordance with the notation for $\beta$ employed earlier, we have $Y_\infty^* = d_e^*$.

2. Axisymmetric Jet. According to [13]

$$\beta = 1.5 \left(\frac{U_e^*}{d_e^*}\right)\left(\frac{X^*}{d_e^*}\right)^{-0,22}.$$

This result is in accordance with the results of experiments also presented in [12]. For the distances to the wall of the nozzle section studied, $\beta \simeq U_e^*/d_e^*$, and consequently, in accordance with the notation employed above for $\beta$, $Y_\infty^* = d_\infty^*/2$. The Reynolds numbers calculated for the $\beta$ values obtained are Re $= 10^4$, Re $= 2 \cdot 10^4$ for plane and axisymmetric cases respectively.

Friction and thermal flux on the wall are defined by the following equations

$$\tau_w = g_1(x) f''(0), \; g_1(x) = F(x); \qquad \text{(plane jet)},$$
$$g_1(x) = (F' - F/x) \qquad \text{(axisymmetric jet)}, \tag{17}$$

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = g_2(x)\,\theta'(0), \; g_2(x) = \sqrt{F'(x)} \qquad \text{(plane jet)},$$
$$g_2(x) = \sqrt{F''(x)} \qquad \text{(axiymmetric jet)}. \tag{18}$$

1. Distribution of Friction. The results obtained for $\tau_w$ were found to be in good agreement with experimental data over the coordinate range x, $0 \le x/R_b \le 0.35$. For $x/R_b \simeq 0.4$, a transition from a laminar form of friction to a turbulent one was observed in the region of viscous mixing. The behavior of calculated and experimental curves for $x/R_b > 0.4$ is quite similar, but there is a difference in the value of friction tension of the order of 50%. Calculation of friction on the wall near the outer limit of the free jet by the function obtained in [15] for a plane jet along a wall

$$\tau_w = 14.4 c_j U_e^2 \left(\frac{x}{d_e} + 11{,}2\right)^{-1,1}; \quad c_f = 1.109 \cdot 10^{-2},$$

shows that the least disagreement of calculated results for a jet interacting with a wall normal to it with data for the jet along the wall is obtained for small values of nozzle–wall distance ($x^*/d_e^* = 2.0$). In the limit as this distance is decreased a degeneration of the free jet into a semibounded jet along a wall occurs.

For a sample evaluation of the value of "apparent" turbulence tension in the viscous mixing layer we assume that the friction tension in sections along the normal to the wall is constant. In this case (Fig. 2) the mean apparent tension, $\overline{\rho u'v'}/u_\infty^2$, obtained on the basis of comparison of calculated results with experimental data [3] is a value which is several times as great as the corresponding value on the boundary layer of a lamina over which a parallel flow passes (according to [16], $\overline{\rho u'v'}/u_\infty^2 \simeq 0.0015$). This may be explained by the fact that, in the case examined here, the interaction of the jet with the wall outside the viscous mixing zone is a jet flow with an increased level of turbulent pulsation in comparison to the typical boundary layer.

2. Thermal Flux. The calculated values of thermal flux ($q_W$) together with experimental data of several authors are presented in Figs. 3 and 4.

A) $X^*/d_e^* = 2.0$. For an axisymmetric jet the difference between the calculated value of $q_W$ and that obtained experimentally in [8] is of the order of 25%. It should be noted that the values presented in [8] are the thermal flux averaged over the surface of the wall. From the experiment of [5] it follows that local thermal flux over the wall has a more complex character than that predicted theoretically. Thus, according to the data of [5] there exists a minimum in thermal flux at the braking point, as well as secondary peaks in the region of spreading over the wall. The mean value of the deviation between calculated and experimental results in 30-50%. Comparison of the results of the present analysis with the calculated data presented in [13] shown that the departure may be as great as a factor of two. In the case of a plane jet, calculated results agree relatively well with the experimental data of [6]. The deviation between calculation and the results of experiment in [9] can be explained by the fact that the latter are averaged values.

B) $X^*/d_e^* = 8.0$. For an axisymmetric jet, the thermal flux distribution calculated in [18] is in satisfactory qualitative agreement with the data of experimental studies [4, 5]. Quantitatively, the deviation between calculated and experimental data varies from 30 to 100%. The data for a plane jet were found to be in good agreement with experimental data [6].

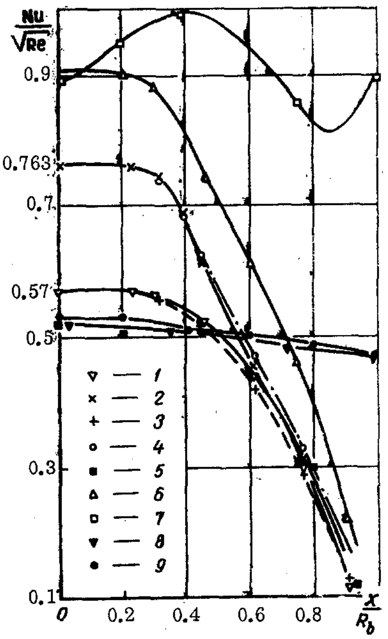


Fig. 3

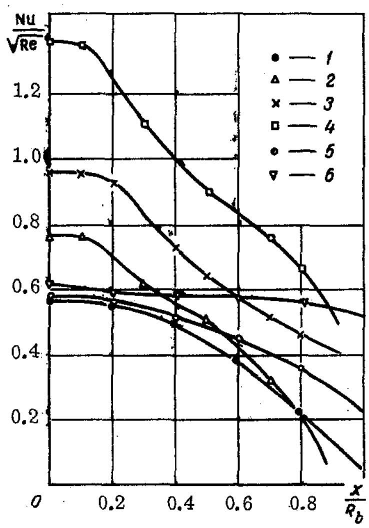


Fig. 4

Fig. 3. Thermal flux distribution in interaction region for $X^*/d_e^* = 2$. Calculation: 1) plane jet; 2) axisymmetric jet; 3) plane jet, $\alpha = -1.0$; 4) axisymmetric jet, $\alpha = -1.0$; Experiment: 5) plane jet [6]; 6) axisymmetric jet [8]; 7) axisymmetric jet [5]; 8) plane jet [9]; 9) plane jet [14].

Fig. 4. Thermal flux distribution in interaction region for $X^*/d_e^* = 8$. Calculation: 1) plane jet, $\alpha = 0$; 2) axisymmetric jet, $\alpha = -1.0$. Experiment: 3) axisymmetric jet [5]; 4) axisymmetric jet [4]; 5) plane jet [6]; 6) plane jet [14].

From the comparison conducted it follows that the effect of turbulent pulsations peculiar to free jets on heat transfer with the wall are reflected to a significantly higher degree with an axisymmetric jet than with a plane jet. The increase in disagreement between calculation and experiment for the axisymmetric jet can be explained by the greater propensity of the axisymmetric jet toward self-preservation of turbulent pulsations. Deviations among the experimental data obtained by different authors under approximately the same flow modes are evidently connected with neglect of the initial level of pulsations at the nozzle section in conducting the experiment.

## NOTATION

| | |
|---|---|
| $x$ | is the coordinate along wall; |
| $y$ | is the coordinate normal to wall; |
| $x_0$ | is the half-width of jet potential core; |
| $\delta$ | is the boundary layer thickness of free jet; |
| $d_e$ | is the width (diameter) of nozzle output section; |
| $R_b$ | is the radius of free jet; |
| $Y_\infty$ | is the distance from wall characterizing the limit of jet interaction with wall; |
| $u$, $v$ | are the velocity components along axes x and y respectively; |
| $T$ | is the temperature; |
| $\Delta T = T - T_a$; | |
| $\Delta T_m = T_m - T_a$; | |
| $\Delta T_e = T_e - T_a$; | |
| $\Omega$ | is the turbulence intensity; |
| $\rho$ | is the density; |
| $\tau_w$ | is the shear stress on wall; |
| $\tau_w^* = \rho\beta\nu\tau_w$; | |
| $\beta$ | is the velocity constant in vicinity of braking point; |
| $\nu$ | is the kinematic viscosity; |
| $Re = u_m^* d_e^* / \nu$ | is the Reynolds number; |
| $Nu = d_e^* (\partial T^* / \partial y^*)_w / (T_m^* - T_w^*)$ | is the Nusselt number; |
| $Pr = \rho c_p \nu / k$ | is the Prandtl number; |
| $F$, $G$ | are the functions determined by equations (4), (8) for plane jet and (11), (14) for axisymmetric jet; |
| $f$, $\theta$ | are the functions determined by Eqs. (9, 10) for plane jet and Eqs. (15), (16) for an axisymmetric jet; |
| $\alpha = (T_w^* - T_a^*) / (T_m^* - T_w^*)$; | |
| $-\rho\overline{u'v'}$ | is the apparent turbulent shear stress in interaction region. |

### Subscripts

| | |
|---|---|
| w | denotes the wall; |
| *a* | denotes the surrounding medium; |
| * | denotes physical values; |
| m | denotes values along the jet axis; |
| e | denotes values in the nozzle exit section. |

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